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## Investigation of the bottomonium ground state $\eta_b$ via its inclusive charm decays

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## Abstract

Based on the non-relativistic QCD factorization formalism, we calculate the bottomonium ground state,  $\eta_b$ , inclusive charm decays at the leading order in the strong coupling constant  $\alpha_s$  and quarkonium internal relative velocity  $v$ . The inclusive charm pair production in  $\eta_b$  decay is mainly realized through  $\eta_b \rightarrow c \bar{c} g$  process, where the charm and anti-charm quarks then dominantly hadronize into charmed hadrons. The momentum distributions of the final states are presented. In this work, we also calculate the  $J/\psi$  inclusive production rate in the  $\eta_b$  decays, where the color-octet contribution is found to be very important. We expect this study may shed some light on finding  $\eta_b$  or knowing more about its nature.

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## I. INTRODUCTION

In high energy research, heavy quarkonium physics is one of the most interesting fields and it plays an important role in understanding the hadron configuration and the microcosmos. Theoretically, due to the non-relativistic nature of heavy quarkonia, it is convenient to research their properties in the framework of non-relativistic potential model and non-relativistic QCD [1], or other theories which work well in the non-relativistic limit. Experimentally, heavy quarkonia have relatively high production rate in both electronic and hadronic collisions and the vector members can be easily seen through their bi-lepton decays. Recently, some new resonances have been observed in the charmonium energy region [2], which enrich the heavy quarkonium spectroscopy and make heavy-quarkonium physics more interesting.

After the spin-triplet bottomonium  $\Upsilon$  was discovered three decades ago, its pseudoscalar partner  $\eta_b$  had been looked for in various experiments. Unfortunately, there is still no conclusive evidence that this elusive particle has been found. As a solid prediction from the quark model, the existence of  $\eta_b$  is indubitable, but its mass and decay channels remain undetermined experimentally at the present time. To search  $\eta_b$ , several experiments have been conducted both in  $e^+e^-$  collisions at the CLEO and the LEP, and hadronic collisions at the Fermilab Tevatron. The advantage of  $e^+e^-$  collisions is its clear background, which is impaired by the fact that production rates for spin-singlet states are generally small. Based on the  $2.4 \text{ fb}^{-1}$  data taken at the  $\Upsilon(2S)$  and  $\Upsilon(3S)$  resonances, CLEO has searched distinctive single photons from hindered  $M1$  transitions of  $\Upsilon(2S)$  and  $\Upsilon(3S)$  to  $\eta_b\gamma$ , and also from the cascade decay  $\Upsilon(3S) \rightarrow h_b\pi^0$ ,  $h_b\pi^+\pi^-$  followed by  $E1$  transition  $h_b \rightarrow \eta_b\gamma$ , but no signals have been found [3]. In the experiments at LEP II, the  $\eta_b(1S)$  is expected to be produced in two-photon process. The ALPHA Collaborations analyzed the  $\gamma\gamma$  interaction data, but found no evident signal in the four- and six-charged-particle final states [4]. The results of the L3 Collaboration and the DELPHI Collaboration were also negative and the upper limits of the products  $\Gamma_{\gamma\gamma} \times Br(\eta_b)$  were set [5, 6, 7]. In comparison to  $e^+e^-$  experiments, the hadronic collision at the Fermilab Tevatron gives large  $\eta_b$  production rate, but due to the complicated hadronic interaction background, the search for  $\eta_b$  there is also hard. Using the full 1992-96(Run I) data, the CDF Collaboration searched the  $\eta_b$  through its exclusive decay to double  $J/\psi$  and found some oblique evidences,

but far from conclusive [8]. Right now, further efforts is being pursued in the Run II data there.

Considering the situation of  $\eta_b$  experiments, theoretical research on its properties is still necessary, such as its mass, the production cross-sections at different colliders, and its various decay channels. Among all properties, the  $\eta_b$  mass is believed to be predicted by potential model, effective theory, and Lattice calculation without much ambiguity, which is very important for experimental observation. Recent theoretical work fixes the  $\Upsilon - \eta_b$  mass splitting in the range of 40 – 60 MeV [9, 10, 11, 12]. In Ref. [13], Braaten, Fleming and Leibovich calculated the  $\eta_b$  production cross section at the Fermilab Tevatron in the framework of NRQCD and evaluated the branching ratio of the decay  $\eta_b \rightarrow J/\psi J/\psi$ . They suggested that  $\eta_b$  should be observable through this decay because of its large branching ratio of  $7 \times 10^{-4 \pm 1}$ . In Ref. [14], Maltoni and Polosa also evaluated the observation potential for  $\eta_b$  at the Tevatron, but they found and suggested that the decay  $\eta_b \rightarrow D^* D^{(*)}$  might be the most optimistic channel to observe  $\eta_b$  signal. In Ref. [15], the relativistic correction to the  $\eta_b \rightarrow J/\psi J/\psi$  decay process was calculated and a much smaller branching ratio in comparison to Ref. [13] was obtained. The author also discussed the  $\eta_b \rightarrow D^* D^{(*)}$  process and got a smaller rate than what in Ref. [14]. However, Santorelli finds that the final state interaction may enhance  $\eta_b \rightarrow J/\psi J/\psi$  decay width by about two orders of magnitude [16]. Instead of  $\eta_b$  decays into hadronic final states, Hao *et al.* calculated the branching ratio of the  $\eta_b$  radiative decay process, i.e.  $\eta_b \rightarrow \gamma J/\psi$  [17]. They claimed that this channel is also a hopeful one in the  $\eta_b$  hunting.

Comparing to the exclusive process, the inclusive process has a large branching ratio, nevertheless normally also has large uncertainties in pinning down the parent particle. Fortunately, the final state distributions of experimental observables are helpful to the inclusive process for the aim. For instance, recently, the inclusive charm production in the  $\chi_b$  and  $\Upsilon$  decays have been studied in [18] and [19] respectively.

At the leading order in  $\alpha_s$  and non-relativistic expansion, the  $\eta_b$  inclusive open charm decay happens through  $b \bar{b} \rightarrow g g^*$  followed by  $g^* \rightarrow c \bar{c}$ , where the initial  $b\bar{b}$  is configured in color-singlet. Based on the result of  $b \bar{b} \rightarrow g c \bar{c}$ , we can roughly estimate the branching fraction of  $b \bar{b} \rightarrow X +$  charmed hadrons. We calculate this process in the framework of NRQCD factorization formalism, in which the un-calculable

nonperturbative effects are represented by the matrix elements of NRQCD operators. According to NRQCD, the color-octet configurations appear as higher order Fock states in  $\eta_b$  decays, which are suppressed by orders of the small magnitude in relative velocity  $v^2$ . Hence, for a leading order calculation, we can safely treat the  $b \bar{b}$  pair inside the  $\eta_b$  to be in color-singlet. However, for the  $J/\psi$  production in  $\eta_b$  decays, although the higher order Fock state, the  $(c\bar{c})(^3S_1^{[8]})$ , is suppressed by  $v^4$  relative to the leading color-singlet configuration, it may be compensated by a factor of  $\alpha_s$  in regarding to color-singlet process.

This paper is organized as follows: the inclusive charm and charmed hadron production in  $\eta_b$  decay is evaluated at leading order in Section II. In Section III, calculation of the process  $\eta_b \rightarrow J/\psi + X$  will be presented. The last section is remained for summary.

## II. THE CHARM QUARK PRODUCTION IN $\eta_b$ DECAYS

In this section we calculate the charm quark production in the inclusive  $\eta_b$  decays. At the leading order in  $v$ , according to the NRQCD factorization formulism, its open charm decay width takes the form

$$\Gamma[\eta_b \rightarrow c \bar{c} + X] = C_1^c \frac{\langle \mathcal{O}_1(^1S_0) \rangle_{\eta_b}}{m_b^2}, \quad (2.1)$$

where  $\langle \mathcal{O}_1(^1S_0) \rangle$  is a NRQCD matrix element, which represents the long-distance effect and gives the probability for finding the heavy quark and antiquark in specific configuration within the meson, and can be evaluated by nonperturbative method such as lattice simulation. The dimensionless short-distance coefficient  $C_1^c$  can be calculated in pQCD. The dominant source of  $C_1^c$  comes from the decay of a color-singlet  $b\bar{b}(^1S_0)$  pair into  $g g^*$ , followed by  $g^* \rightarrow c \bar{c}$ .

We can calculate the process in the nonrelativistic limit, in which the  $b\bar{b}(^1S_0)$  pair can be taken as no relative momentum within  $\eta_b$ , i.e.,  $p_b = p_{\bar{b}} = Q/2$ , where  $Q$  is the momentum of the  $\eta_b$ . In this situation, for the  $b\bar{b}$  pair to form  $\eta_b$ , when it is in a color-singlet state, one can replace the product of the Dirac spinors for  $b$  and  $\bar{b}$  in the initial state with the projector:

$$u(p_b) \bar{v}(p_{\bar{b}}) \longrightarrow \frac{1}{2\sqrt{4\pi}} (\not{Q} + M_{\eta_b}) i\gamma_5 \times \left( \frac{1}{\sqrt{M_{\eta_b}}} R_{\eta_b}(0) \right) \otimes \left( \frac{\mathbf{1}_c}{\sqrt{N_c}} \right), \quad (2.2)$$

where  $N_c = 3$ , and  $1_c$  stands for the unit color matrix.  $M_{\eta_b}$  is the masses of  $\eta_b$ . At leading order in non-relativistic expansion, it could be understood that  $M_{\eta_b} \approx 2 m_b$ . The nonperturbative parameters,  $R(0)_{\eta_b}$  are color-single radial wave functions at the origin for  $\eta_b$ , which can be either reached from phenomenological potential models or directly extracted from experiments. The relation between the  $R(0)_{\eta_b}$  and  $\langle \mathcal{O}_1^{\eta_b}(^3S_1) \rangle$  reads  $\langle \mathcal{O}_1^{\eta_b}(^3S_1) \rangle = (N_c/2\pi) |R(0)_{\eta_b}|^2 (1 + O(v^4))$ .

One can then get the partial decay width straightforwardly for the process  $b\bar{b}(^1S_0) \rightarrow c(p_1) + \bar{c}(p_2) + g(k)$ . That is,

$$d\Gamma[b\bar{b}(^1S_0) \rightarrow c\bar{c}g] = \frac{1}{2M_{\eta_b}} \sum_{ccg} |M_{str}|^2 d\Phi_3, \quad (2.3)$$

where  $M_{str}$  is the amplitude for the process and the  $\Phi_3$  represents the three-body phase space, which shows

$$d\Phi_3 = \frac{1}{(2\pi)^9} \cdot \frac{d^3p_1}{2E_1} \cdot \frac{d^3p_2}{2E_2} \cdot \frac{d^3k}{2k_0} \cdot (2\pi)^4 \delta^4(Q - p_1 - p_2 - k). \quad (2.4)$$

In the initial state rest frame, after integrating over the variables which are independent of the amplitude, the phase space integration can be further simplified as

$$d\Phi'_3 = \frac{1}{(2\pi)^3} \frac{1}{4M_{\eta_b}} dE_1 dS_{13}. \quad (2.5)$$

Here,  $S_{13} = p_1 \cdot k$ .

In the numerical calculation, we take  $m_b = 4.65 \pm 0.15$  GeV,  $m_c = 1.50 \pm 0.05$  GeV, and  $\alpha_s(m_b) = 0.22$ . The magnitude of the radial wave function at the origin  $R(0)$  of  $\eta_b$  equals approximately to that of its spin triplet partner  $\Upsilon$ , which can be determined from the experimental data on the decay width of  $\Gamma(\Upsilon \rightarrow e^+e^-) = (1.340 \pm 0.018) \times 10^{-6}$  GeV [20]. That is:  $|R(0)_{\eta_b}|^2 = |R(0)_{\Upsilon}|^2 = 4.89 \pm 0.07$  GeV<sup>3</sup>. With these inputs and by varying the strong coupling scale from  $m_b/2$  to  $2m_b$ , for the aim of error estimation, we have

$$\Gamma[b\bar{b}(^1S_0) \rightarrow c\bar{c}g] = 190.8^{+190.0}_{-86.3} \text{ KeV}. \quad (2.6)$$

Since at leading order the total decay width of  $\eta_b$  is

$$\Gamma_{tot}(\eta_b) \approx \Gamma(\eta_b \rightarrow gg) = \frac{8}{3} \frac{\alpha_s^2}{M_{\eta_b}^2} |R_{\eta_b}(0)|^2, \quad (2.7)$$

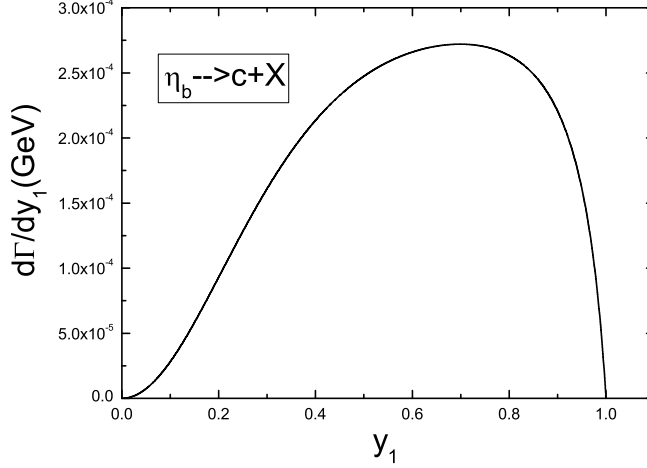


FIG. 1: The decay rate variation over momentum fraction  $y_1$  in the inclusive process  $\eta_b \rightarrow c + X$ , by taking the central values of inputs.

the branching ratio hence is readily obtained to be

$$Br[b\bar{b}(^1S_0) \rightarrow c\bar{c}g] = 2.6_{-0.6}^{+0.9} \times 10^{-2} . \quad (2.8)$$

For inclusive decay process, giving out the experiment observable differential distribution will be useful. For this purpose, We define two fractions  $x_1 = E_1/E_b$  and  $r_c = m_c^2/m_b^2$ , where  $E_1$  and  $E_b$  stand for the energy of charm quark and bottom quark, and in nonrelativistic approximation the  $E_b = m_b = M_{\eta_b}/2$ . The region of variable  $x_1$  is  $\sqrt{r_c} < x_1 < 1$ . In some cases, instead of  $x_1$  it is convenient to use another variable  $y_1$ , which is the momentum of the charm quark divided by its kinematically allowed maximum value in  $\eta_b$  decays [18, 19]. The relation between these two variables reads

$$x_1 = \sqrt{(1 - r_c)y_1^2 + r_c} , \quad (2.9)$$

$$y_1 = \sqrt{\frac{x_1^2 - r_c}{1 - r_c}} . \quad (2.10)$$

The range of  $y_1$  is  $0 < y_1 < 1$ . Figure 1 exhibits the decay rate distribution over the momentum fraction  $y_1$ .

Because almost all the charm quarks may eventually hadronize into charmed hadrons, like  $D^0$ ,  $D^\pm$ ,  $D_s$ , and  $\Lambda_c$ , etc., we schematically show the  $D^+$  meson differential distribution in  $\eta_b$  decays in the fragmentation approximation, similar as done

in Refs. [18, 19]. It is well-known that the fragmentation function  $D_{c \rightarrow h}(z)$  represents the probability of a charm quark fragmenting into the charmed hadron  $h$ . Here, the  $z$  is a Lorentz boost invariant variable, defined as  $z = \frac{E_h + p_h}{E_1 + p_1}$ . In practice calculation, we will simply neglect the difference between fragmenting charm quark mass and the charmed hadron mass. The  $z$  can be reexpressed as:

$$z = \frac{z_h}{z_1} \quad (2.11)$$

with

$$z_1 = \frac{\sqrt{(1-r_c)y_1^2 + r_c} + y_1\sqrt{1-r_c}}{1 + \sqrt{1-r_c}}, \quad (2.12)$$

$$z_h = \frac{\sqrt{(1-r_c)y_h^2 + r_c} + y_h\sqrt{1-r_c}}{1 + \sqrt{1-r_c}}. \quad (2.13)$$

Then the momentum distribution of the charmed hadron can be expressed as [18, 19]

$$\frac{d\Gamma}{dy_h} = \frac{dz_h}{dy_h} \int_{z_h}^1 \frac{dz_1}{z_1} D(z_h/z_1) \frac{dy_1}{dz_1} \frac{d\Gamma}{dy_1}, \quad (2.14)$$

According to the Kartvelishvili-Likhoded-Petrov (KLP) [21] fitting for fragmentation function

$$D_{c \rightarrow h}(z) = N_h z^{\alpha_c} (1-z). \quad (2.15)$$

Here, by using the optimal value of  $\alpha_c = 4$  for  $D^+$  fitted by the Belle Collaboration [22], one gets the normalization coefficient  $N_h = 8.04$  [18].

We show in Figure 2 the  $D^+$  meson differential production distribution in  $\eta_b$  decays in the fragmentation approximation. For other charmed hadrons, the corresponding distributions can be obtained similarly.

It should be mentioned that since the NRQCD factorization breaks down in the  $y_1 \rightarrow 1$  limit, the velocity and coupling expansions are no more valid in the vicinity of the end point. A proper treatment for this endpoint illness is to resum the large logarithms of  $\log(1-y_1)$  [23, 24, 25, 26, 27, 28] and invoke the shape function [29]. For these kinds of content readers should refer to the related references in the literature; and in this situation, the final state distribution results at the endpoint in this work should not be taken seriously. Roughly speaking, when  $y_1 < 0.7$  the endpoint effects become weak and our predictions turn to be robust [30]. Fortunately, for total decay



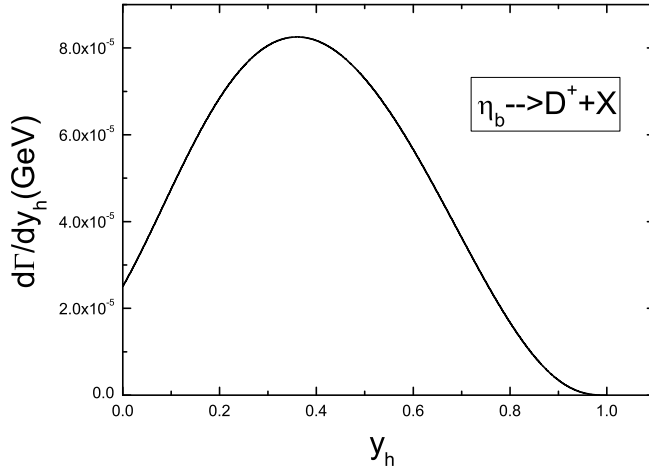


FIG. 2: The decay rate variation over momentum fraction  $y_h$  in the inclusive process  $\eta_b \rightarrow D^+ + X$  with central values of inputs.

widths, the endpoint influence is minor and the predictions are quite reliable. For the charmed hadron production, apart from the upper point problem, the lower limit also poses a problem for the fragmentation approximation, although (2.15) only comes from the phenomenological fitting. Therefore, the lower endpoint prediction should not be taken seriously as well. In Ref.[18], there are detailed discussions about the validity of the fragmentation approximation.

### III. $\eta_b$ INCLUSIVE DECAY TO $J/\psi + X$

In this section, we present the calculation of  $J/\psi$  inclusive production and its momentum distribution in the  $\eta_b$  decays, as shown in Figure 3. As mentioned in above, at the leading order in  $v$  only the color-singlet  $b\bar{b}(^1S_0)$  contribution in the initial state is the necessary ingredient to be considered, since the higher Fock state contribution can not get big enhancement from kinematic or dynamic reasons. However, for the final state  $J/\psi$  both the color-singlet and color-octet effects should be included, because the latter one can get one  $\alpha_s$  and kinematic compensation in comparison with the color-singlet process. Since it is the  $b\bar{b}$  pair annihilation in the initial state and the creation of the  $c\bar{c}$  pair, as well as the emission of gluon or quarks, takes place at the hard scales

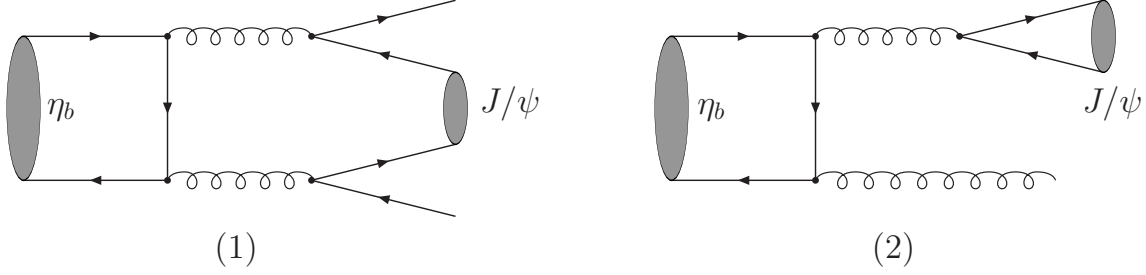


FIG. 3: Lowest-order diagrams that contribute to the inclusive process:  $\eta_b \rightarrow J/\psi X$ . The  $J/\psi$  in Diagrams (1) is a color-singlet state, while the one in (2) is a color-octet state.

set by the heavy quark masses, it is legitimate to tackle this semi-inclusive process in the pQCD framework.

The NRQCD formalism allows the systematic calculation of inclusive cross sections for quarkonium decays in perturbation QCD to any order in  $\alpha_s$  and  $v^2$ , where  $v$  is the typical relative velocity of the heavy quark inside the quarkonium. Using the NRQCD velocity scaling rules [1], we know the color-singlet process, the Figure 3(1), is at order of  $\alpha_s^4 v^3$ , since the matrix element  $\langle O_1^V(^3S_1) \rangle$  is of order  $v^3$ . Whereas, the color-octet process, the Figure 3(2), is at order  $\alpha_s^3 v^7$ . Here,  $v$  denotes the heavy quark relative velocity in Charmonium system. Potential model calculation indicates that the average value of  $v^2$  is about 0.3. And, the QCD coupling constant  $\alpha_s(m_b) \approx 0.22$ . In addition of the  $\alpha_s$  compensation, the color-octet mechanism may also enhanced by the single gluon propagator. Our following calculation really shows that the color-octet process should be included in this calculation.

The decay width can be formulated as

$$\Gamma[\eta_b \rightarrow J/\psi + X] = A_1 \langle O_1^{J/\psi}(^3S_1) \rangle + A_2 \langle O_8^{J/\psi}(^3S_1) \rangle, \quad (3.1)$$

where, the  $A_1$  and  $A_2$  are perturbative calculable short-distance coefficients. We first calculate the color-singlet coefficient. It is customary to start with the parton process; here it is  $b(p_b) \bar{b}(p_{\bar{b}}) \rightarrow c(p_c) \bar{c}(p_{\bar{c}}) + c(k_1) + \bar{c}(k_2)$ . Then project the matrix element onto corresponding color-singlet configurations. For the  $b\bar{b}$  in initial state, it is the same as in the last section. For the  $J/\psi$  production, the color-singlet projector is

$$v(p_{\bar{c}}) \bar{u}(p_c) \longrightarrow \frac{1}{2\sqrt{4\pi}} \not{\epsilon}_{J/\psi}^* (\not{P} + M_{J/\psi}) \times \left( \frac{1}{\sqrt{M_{J/\psi}}} R_{J/\psi}(0) \right) \otimes \left( \frac{\mathbf{1}_c}{\sqrt{N_c}} \right), \quad (3.2)$$

where  $\epsilon_{J/\psi}^\mu$  is the  $J/\psi$  polarization vector satisfying  $\epsilon_{J/\psi}(\lambda) \cdot \epsilon_{J/\psi}^*(\lambda') = -\delta^{\lambda\lambda'}$  and  $P \cdot \epsilon_{J/\psi} = 0$ .  $R_{J/\psi}(0)$  is the radial wave function at the origin, which is also valued through  $J/\psi$  leptonic decay width. Combining all these together, one can easily get the  $\eta_b$  to  $J/\psi + c + \bar{c}$  decay amplitude for the color-single case, i.e.

$$\begin{aligned}
M_{str}^1 &= C_1 g_s^4 \frac{R_{\eta_b}(0) R_{J/\psi}(0)}{4\pi \sqrt{M_{J/\psi} M_{\eta_b}}} \\
&\times Tr[(\not{Q} + M_{\eta_b}) \gamma_5 \gamma_\mu ((k_2 - k_1) \cdot \gamma + M_{\eta_b}) \gamma_\nu] \\
&\times \frac{1}{(k_2 - k_1)^2 - M_{\eta_b}^2} \times \frac{1}{(k_1 + P/2)^2} \times \frac{1}{(k_2 + P/2)^2} \\
&\times \bar{u}(k_1) \gamma^\mu \not{\epsilon}_{J/\psi}(\not{P} + M_{J/\psi}) \gamma^\nu v(k_2), \quad (3.3)
\end{aligned}$$

where  $C_1$  is the corresponding color factor.  $k_1$  and  $k_2$  are the momenta carried by the external charm quark and anti-quark, respectively.

Next, we present the calculation for color-octet process. At the parton level it is  $b(p_b) \bar{b}(p_{\bar{b}}) \rightarrow c(p_c) \bar{c}(p_{\bar{c}}) + g(k)$  process followed by projecting the  $c \bar{c}$  spinors onto the color-octet configuration,  $^3S_1^{[8]}$ , while keeping on configuring the initial  $b \bar{b}$  in color-singlet. The color-octet projector is

$$v(p_{\bar{c}}) \bar{u}(p_c) \longrightarrow \frac{1}{2\sqrt{4\pi}} \not{\epsilon}_{J/\psi}^* (\not{P} + M_{J/\psi}) \times \left( \frac{1}{\sqrt{M_{J/\psi}}} R_{J/\psi}^8(0) \right) \otimes \sqrt{2} T_{ij}^a. \quad (3.4)$$

Here,  $T_{ij}^a$  denotes the SU(3) generator. And, we introduce another phenomenological parameter  $R_{J/\psi}^8(0)$ , which stands for the color-octet nonperturbative effect. The relation between  $R_{J/\psi}^8(0)$  and the NRQCD matrix element  $\langle \mathcal{O}_8^{J/\psi}(^3S_1) \rangle$  is defined as:

$$\langle \mathcal{O}_8^{J/\psi}(^3S_1) \rangle = \frac{3N_c}{2\pi} |R_{J/\psi}^8(0)|^2. \quad (3.5)$$

From the fitted value of  $\langle \mathcal{O}_8^{J/\psi}(^3S_1) \rangle \approx 1.5 \times 10^{-2} \text{GeV}^3$  [31], we have  $|R_{J/\psi}^8(0)| = 0.102 \text{ GeV}^{3/2}$ . Then the decay amplitude for color-octet case is

$$\begin{aligned}
M_{str}^{8(a)}(\lambda_1, \lambda_2) &= C_8 g_s^3 \frac{R_{\eta_b}(0) R_{J/\psi}^8(0)}{4\pi \sqrt{M_{J/\psi} M_{\eta_b}}} \\
&\times Tr[(\not{Q} + M_{\eta_b}) \gamma_5 \not{\epsilon}_g^a(\lambda_2) ((Q/2 - k) \cdot \gamma + M_{\eta_b}/2) \gamma^\nu] \\
&\times \frac{1}{M_{J/\psi}^2} \times \frac{1}{M_{\eta_b}^2 - M_{J/\psi}^2} \times Tr[\not{\epsilon}_{J/\psi}(\lambda_1) (\not{P} + M_{J/\psi}) \gamma_\nu], \quad (3.6)
\end{aligned}$$

where  $C_8$  is the color factor,  $k$  is the momentum carried by the external gluon, and  $\epsilon_g^\mu$  is the gluon polarization satisfying  $k \cdot \epsilon_g = 0$ .

With the matrix elements  $M_{str}^1$  and  $M_{str}^8$  we can immediately get the  $\eta_b \rightarrow J/\psi + X$  decay width. The analytical result for it is a bit lengthy, and will not be presented here. For our numerical estimation, the non-relativistic limit is also enforced for the charmonium. That is we take the  $M_{J/\psi} \approx 2m_c$  approximation. From  $\Gamma(J/\psi \rightarrow e^+e^-) = (5.55 \pm 0.14) \times 10^{-6} \text{ GeV}$  [20], we get  $|R_{J/\psi}(0)|^2 = 0.527 \pm 0.013 \text{ GeV}^3$ . Then, the decay width for the concerned process reads,

$$\Gamma(\eta_b \rightarrow J/\psi_{color-singlet} + X) = 0.13_{-0.08}^{+0.26} \text{ KeV} , \quad (3.7)$$

$$\Gamma(\eta_b \rightarrow J/\psi_{color-octet} + X) \cong 2.16 \text{ KeV} , \quad (3.8)$$

$$\Gamma_{total}(\eta_b \rightarrow J/\psi + X) \cong 2.29 \text{ KeV} . \quad (3.9)$$

That means that the  $\eta_b \rightarrow J/\psi + X$  process has a branching ratio of  $3.23 \times 10^{-4}$  or so in the  $\eta_b$  decays. Here, the uncertainty estimate of the color-singlet process is performed in the same way as in the preceding section. Whereas, considering of the large uncertainties remaining in the color-octet matrix element fitting, we carry the numerical calculation for color-octet process by only taking the central values of the inputs.

Like in section II, to give out the differential decay width we define three new variables, the  $x_2$ ,  $r_{J/\psi}$ , and  $y_2$ , as

$$x_2 = E_{J/\psi}/E_b , \quad (3.10)$$

$$r_{J/\psi} = M_{J/\psi}^2/m_b^2 , \quad (3.11)$$

$$y_2 = \sqrt{\frac{x_2^2 - r_{J/\psi}}{1 - r_{J/\psi}}} . \quad (3.12)$$

Then we can express the partial decay width of  $\eta_b \rightarrow J/\psi_{color-singlet} + X$  as

$$d\Gamma[\eta_b \rightarrow J/\psi_{color-single} + X] = \frac{1}{2M_{\eta_b}} \sum_{J/\psi c\bar{c}} |M_{str}^1|^2 d\Phi_3 \quad (3.13)$$

In analogy to what is performed in the last section, we get the momentum distribution  $d\Gamma(\eta_b \rightarrow J/\psi_{color-singlet} + X)/dy_2$ , as shown in Figure 4. For the the process  $\eta_b \rightarrow J/\psi_{color-octet} + X$ ,

$$d\Gamma[\eta_b \rightarrow J/\psi_{color-octet} + X] = \frac{1}{2M_{\eta_b}} \sum_{\lambda_1, \lambda_2} |M_{str}^8(\lambda_1, \lambda_2)|^2 d\Phi_2 . \quad (3.14)$$

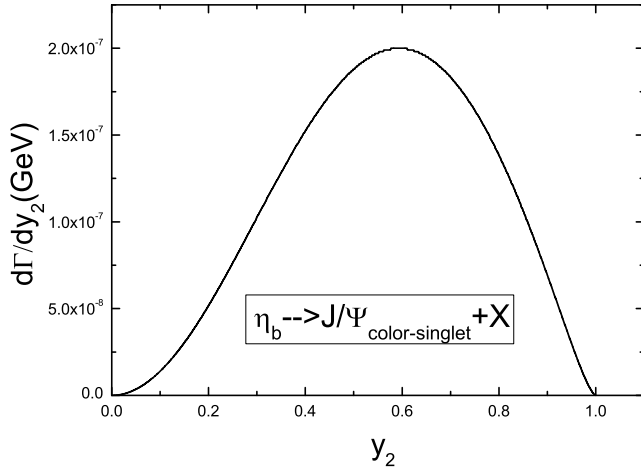


FIG. 4: The  $J/\psi$  momentum distribution in the inclusive process  $\eta_b \rightarrow J/\psi_{color-singlet} + X$ .

Since this is a two-body decay process, the  $J/\psi$  momentum distribution  $d\Gamma(\eta_b \rightarrow J/\psi_{color-octet} + X)/dy_2$  is only a delta function peaked at  $y_2 = \sqrt{\frac{(M_{\eta_b}^2 - M_{J/\psi}^2)^2}{4M_{\eta_b}^2(M_{\eta_b}^2 - 4M_{J/\psi}^2)}} = 0.6$ . Again, for the  $\eta_b$  to  $J/\psi$  inclusive decay distribution, one should pay attention to the endpoint problem [32]. In particular for the color-octet contribution, the  $\eta_b$  two-body decay at leading order resulting in a delta function distribution, which is smeared out by the non-perturbative effects and resulting in a shape function [33].

The numerical result shows that the branching ratio for color-octet process is larger than the one for color-singlet process by about an order, which offers an opportunity to check the existence of color-octet mechanism experimentally. Considering of the uncertainties existing in the magnitude of color-octet matrix element, the numerical difference between these two processes might shrink down, nevertheless, they give a distinctively different momentum distribution, which may also help experimenters to distinguish them in the future experiment.

#### IV. SUMMARY

We have studied in the framework of NRQCD the inclusive charm production in the decay of the pseudoscalar bottomonium state  $\eta_b$ . We find that it gives a quite

large branching fraction. Since the produced charm quarks will dominantly evolve into charmed hadrons, by employing the KLP fragmentation function, we give out the momentum distribution of  $D^+$ , as an example.

We have also calculated the decay width and the momentum distribution of the inclusive  $J/\psi$  production in the  $\eta_b$  decay. We find that in this case the color-octet process should be taken into consideration. However, this two different  $J/\psi$  production schemes have obviously different momentum distributions. This is a distinct character of this process, which will be helpful for future experiment to investigate the  $\eta_b$  and to the study  $J/\psi$  production.

In all, to investigate the elusive  $\eta_b$  is still an interesting task for both theory and experiment. Our explicit calculation shows that  $\eta_b$  inclusive decays to charm pair(in experiment the charmed hadron pair) and  $J/\psi$  have quite large branching fractions. These processes can be helpful for people to hunt for the  $\eta_b$  at the Fermilab Tevatron, or LHC, where copious  $\eta_b$  are expected.

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